This completes the proof of the theorem formulated in Sec. 2.

For a complete solution of the piston problem, it remains to construct a solution of the mixed problem for Eqs. (1.1) with the impermeability condition on Γ' and the condition of continuous touching to the centered wave on the characteristic Γ_2. Here the conditions for the consistency of the given boundary value problems are already satisfied. We note that the result obtained can also be used in problems of describing the interaction of strong discontinuities. The case when the surface γ_0 lies in the hyperplane $t = 0$ in E^4 is examined in [4]. Such centered waves arise in describing the decomposition of an arbitrary discontinuity on a curvilinear surface [5].

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INJECTION EFFECT IN A CONTAINED EXPLOSION IN A LIQUID-SATURATED MEDIUM

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The escape of gaseous products from the cavity of a contained explosion into dry rock has been considered [1, 2]. The escape is due to the residual elevated pressure in the cavity. However, a contained explosion in a liquid-saturated medium may result in an elevated pore pressure around the explosion cavity, which exceeds the pressure in the cavity itself. This is possible because the fluid in the pores is compressed when the shock wave passes, and the strength of the skeleton means that the pressure does not revert to the initial value on unloading: there remains a residual pressure in the pores of the order of the strength of the skeleton. On the other hand, the pressure in the explosion cavity at the end of the explosion is close to the lithostatic pressure, i.e., below the pressure in the pores. Therefore, the elevated pore pressure may cause implosion, namely injection of liquid into the cavity. This alters the temperature and pressure within the cavity, which in turn influences the cavity collapse.

Here we consider theoretically the implosion effect and the influence on the heat and mass transfer on explosion in a liquid-saturated medium.

Model for Heat and Mass Transfer after Explosion in a Water-Saturated Medium. It has been pointed out [3] that there may be a rise in the pore pressure after a contained explosion for the model of [4]. Figure 1 shows a typical graph for the pore pressure. The rise in pore pressure after the passage of a shock wave is indirectly confirmed by the ground water-level measurements after explosions [5]. Therefore, an explosion in a saturated rock may result in filtration not from the cavity but into it. When the liquid enters the cavity, where the temperature is about 10^4°K and the pressure about 15 MPa, the liquid evaporates, taking up energy from the rock vapor, which is thereby cooled. When that vapor reaches a state of saturation, it begins to condense and release latent heat. This may raise the

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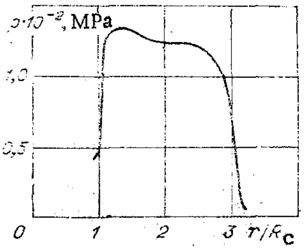

Fig. 1.

pressure in the cavity and result in a nonmonotone course for the pressure in the pore space near it.

Further entry of liquid into the cavity cools it and therefore provides conditions for vapor condensation, which results in a fall in the pressure.

The liquid infiltration outside the cavity is described by the nonstationary equation for infiltration

$$a\Delta p = \partial p/\partial t \tag{1}$$

with the following boundary and initial conditions:

$$p(R_c,\ t) = p_c(t); \tag{2}$$

$$p(\infty,\ t) = p_0; \tag{3}$$

$$p(r,\ 0) = p_0. \tag{4}$$

Here a is the piezoconductivity coefficient; p, pressure of the liquid during the infiltration; R_c, cavity radius; p_0, initial pressure of the liquid after the explosion; and $p_c(t)$, pressure in the cavity; the last is dependent on m(t), the mass of liquid injected into the cavity at time t, which is defined by

$$m(t) = -4\pi R_c^2 \rho \frac{k}{\mu^*} \int_0^t \nabla p(R_c,\ \tau)\, d\tau, \tag{5}$$

where k is the permeability of the medium and ρ and μ^* are, respectively, the density and viscosity of the liquid. The energy-conservation equation defines the temperature of the vapor in the cavity up to the time t_1 at which the vapor begins to condense:

$$E_0 + E_t(t) + m(t)c_l T^+ + \eta(t - t_0)\xi m_0 L_1 = m(t)L_2 + m(t)\frac{c}{\mu_2}T + \frac{1 - \xi\eta(t - t_0)}{\mu_1} m_0 c T. \tag{6}$$

Here t_0 is the instant when the refractory component of the rock begins to condense; E_0, internal energy of the vapor in the cavity at the start of injection; $E_t(t)$, part of the energy dissipated in the rock around the cavity and transported by the liquid infiltrating into the cavity up to time t; m_0, mass of rock evaporated by the explosion; c_l, specific heat of the liquid; c, molar specific heat of the gas; L_1, latent heat of evaporation of the refractory component (SiO_2); L_2, latent heat of evaporation of the liquid; μ_1 and μ_2, molecular masses of SiO_2 and the liquid; T^+, initial temperature of the liquid entering the cavity; T, temperature in the cavity at time t; ξ, proportion of the vapor of the refractory component that has condensed up to time t; $\eta(t - t_0)$, unit Heaviside function; and m(t), amount of liquid infiltrating into the cavity up to time t.

We assume that after t_0 the vapor pressure of the refractory component is described by a saturation curve with parameters p_1 and B_1:

$$p = p_1 \exp\{-B_1/T\}. \tag{7}$$

On the other hand, we assume that for SiO_2 vapor we can apply an ideal-gas equation for a time-varying mass:

$$pV_c = (m^*(t)/\mu_1)RT, \tag{8}$$

where $m^*(t) = m_0(1 - \xi(t))$; from (8) we get

$$\xi(t) = 1 - pV_c\mu_1/(m_0 RT),$$

where p is the partial pressure of the SiO_2 vapor, which is described by (7). After the start of condensation of the liquid vapor, any fresh incoming liquid will not evaporate, and the energy-balance equation will take the form

$$E_T(t) - E_T(t_1) + \Delta m(t) L_2 = c_l\ [m(t) - m(t_1)](T - T^+), \tag{9}$$

where $\Delta m(t)$ is the mass of liquid condensed up to time t.

The pressure in the cavity is determined by the sum of the partial pressures for the rock and liquid vapors:

$$\begin{aligned} p_C(t) = (p_C(0)/T(0))\, T(t)\, \eta(t_0 - t) + p_1 \exp(-B_1/T)\, \eta(t - t_0) + \\ + (m(t)/\mu_2)(RT(t)/V_C)\, \eta(t_1 - t) + p_2 \exp(-B_2/T)\, \eta(t - t_1), \end{aligned} \tag{10}$$

where $V_C = (4/3)\pi R_C^3$ is the volume of the cavity, p_1 and B_1 are the constants in the saturation curve for SiO_2, p_2 and B_2 are the same for the liquid, and R is the universal gas constant. The first term in (10) describes the change in partial pressure of the refractory component before condensation starts, the second describes the partial pressure of the refractory component (SiO_2) during condensation, the third describes the partial pressure of the vapor of the liquid before condensation starts, and the fourth describes the partial pressure of the liquid vapor after the start of condensation.

Infiltration into a Finely Divided Medium. Calculations [3] show that after a contained explosion some 43% of the energy remains in the cavity and about 20% is dissipated in the region of the cavity in a layer whose thickness constitutes about 2% of the cavity radius. The infiltrating liquid takes up heat from this layer and transports it into the cavity. We consider two limiting cases for the heat transfer between the liquid and the heated layer: 1) the temperatures of the pieces of rock and fluid come to equilibrium during the infiltration, and 2) the rate of infiltration is so high that the heat transfer between the liquid and the rock can be neglected. The characteristic heating time of the pieces is $\tau_1 \sim R_0^2/\varkappa$, where R_0 is the radius of a piece and $\varkappa$ is the thermal diffusivity. The characteristic time for heat transfer by a piece of rock to the liquid is $\tau_2 \sim 2R_0\mu^*\nabla p/k$. In the first case ($\tau_1 \leqslant \tau_2$) one can solve (1)-(10) in approximate analytic form, which corresponds to infiltration of a liquid into a finely divided medium with characteristic piece size of $R_0 \leqslant 10^{-3}$ m.

The analytic solution to (1)-(10) can be obtained for the initial stage of infiltration, where the injection has altered the pore pressure only near the cavity and the infiltration of the liquid may be considered as one-dimensional, when (1) becomes

$$a\partial^2 p/\partial x^2 = \partial p/\partial t, \tag{11}$$

where $x = r - R_C$.

Figure 1 shows that the initial pore pressure of the liquid may be taken as constant:

$$p(x, 0) = p_0. \tag{12}$$

The pressure has not had time to change in the initial stage of the process in areas remote from the cavity, so the right-hand boundary condition is

$$p(\infty, t) = p_0. \tag{13}$$

The rate of injection is determined by $(p_0 - p_C(t))$; the pore pressure substantially exceeds the pressure in the cavity during the implosion, so to calculate the amount of injected liquid we neglect the pressure change in the cavity:

$$p(0, t) = p_C(0). \tag{14}$$

This simplification results in a small error in the calculations $\sim\Delta p_C/(p_0 - p_C)$, but it enables one to solve two independent problems instead of a single self-consistent one: liquid infiltration and the changes in pressure and temperature in the cavity. The solution to (11)-(14) is [6]

$$p(x, t) = p_0 - [p_0 - p_C(0)]\, [1 - \Phi(x/2\sqrt{at})], \tag{15}$$

where $\Phi(z) = \frac{2}{\sqrt{\pi}} \int\limits_0^z \exp(-y^2)\, dy$; the dimensions of the heated layer are much less than the characteristic scales for the pressure profile, so the infiltration rate can be taken as constant and equal to the rate at the boundary of the cavity. From (15) we get

$$v_\Phi(R_c) = (k/\mu^*)(p_0 - p_c(0))/\sqrt{\pi a t}. \tag{16}$$

The temperatures of the fluid and the pieces of rock are equal for a finely divided medium, and we have as follows from the energy-conservation equation:

$$v_\Phi c_l \rho \partial W/\partial x = c_r \rho_r (1 - n) \partial W/\partial t, \tag{17}$$

where c_r and ρ_r are the specific heat and density of the rock, n is the porosity, and W is the temperature of the liquid. On the basis of (16), (17) becomes

$$(b/\sqrt{t}) \partial W/\partial x = \partial W/\partial t, \tag{18}$$

where $b = \frac{k}{\mu^*} \frac{p_0 - p_c(0)}{\sqrt{\pi a}} \frac{c_l \rho}{c_r \rho_r (1 - n)}$; the solution to (18) is $W(x, t) = W(x + 2b\sqrt{t})$, and the detailed form of this function is determined by the initial temperature distribution in the heated layer, which can be out as

$$W(x, 0) = W_0 \exp(-Dx/R_c) + T^+,$$

which agrees well with theoretical and experimental data. The heated layer at the start contains the energy

$$E_1 = 4\pi [(1 - n) c_r \rho_r + n c_l \rho] \int_{R_c}^{\infty} (W(x, 0) - T^+) r^2 dr.$$

The energy transported by the liquid from the heated layer to the cavity up to time t is

$$E_T(t) = E_1(1 - \exp(-2bD\sqrt{t}/R_c)). \tag{19}$$

The mass of liquid injected up to time t is defined by (5), which with (16) gives

$$m(t) = 8\pi R_c^2 \rho \frac{k}{\mu^*} \frac{p_0 - p_c(0)}{\sqrt{\pi a}} \sqrt{t}. \tag{20}$$

We substitute (19) and (20) into (6), (9), and (10) to get algebraic equations for the temperature and pressure. Up to t_0, we see from (6) that the temperature is the following function of time:

$$T = \left[E_0 + (c_l T^+ - L_2) 8\pi R_c^2 \rho \frac{k}{\mu^*} \frac{p_0 - p_c(0)}{\sqrt{\pi a}} \sqrt{t} + E_T(t)\right] \Big/ \left[\frac{m_0 c}{\mu_1} + \frac{c}{\mu_2} 8\pi R_c^2 \rho \frac{k}{\mu^*} \frac{p_0 - p_c(0)}{\sqrt{\pi a}} \sqrt{t}\right], \tag{21}$$

where $E_t(t)$ is defined by (19). The instant t_0 is defined as the instant at which the partial pressure of the SiO_2 vapor coincides with the pressure calculated from the saturation curve for SiO_2. It is impossible to obtain an analytic expression for T(t) for $t > t_0$, and therefore (6) is solved by means of successive time steps and iteration. At some value $t' > t_0$, we substitute $m(t')$ and $E_t(t')$ into (6). As a zeroth approximation we take the temperature of the cavity in the preceding time step and substitute everywhere apart from the exponent in $\xi(T)$. We use the explicit form of $\xi(T)$ and take logarithms to get the first approximation for T at time t' and continue the iteration. Usually, the iteration converges very rapidly (2-3 iterations suffice).

These relationships have been used in calculating an implosion with the following parameters (explosion in a water-saturated medium): $R_c = 30$ m; $\rho = 1$ g/cm^3; $a = 10^3$ cm^2/sec; $p_0 = 100$ MPa; $p_c(0) = 15$ MPa; $k = 0.1$ d; $\mu^* = 1$ cP; $E_0 = 3.35 \cdot 10^{12}$ J; $m_0 = 3.35 \cdot 10^5$ kg; $c_l = 4.18$ J/(g·deg); $c = 4.5$ R; $L_2 = 2.26 \cdot 10^3$ J/g; $L_1 = 153 \cdot 10^4$ J/g; $T^+ = 300$°K; $\mu_2 = 18$; $\mu_1 = 60$.

Figure 2 shows the results as the time dependence of the pressure and temperature in the cavity. The solid line shows the temperature and the broken line the pressure. There are three substantially different periods denoted by I, II, and III. Period I corresponds to cooling of the cavity due to implosion before the start of the condensation of the refractory component. Period II begins with the condensation of the refractory component; it is characterized by constant temperature and by a rise in pressure due to the deposition of the latent heat of condensation of the refractory component. Period III begins when much of the refractory component has condensed: the pressure again begins to fall, while the temperature continues to fall. The most notable point is that the implosion effect results in marked cooling of the cavity, with the temperature falling from 10^4 to 3200°K in 30 sec. Another feature related to this is the nonmonotone variation of the pressure.

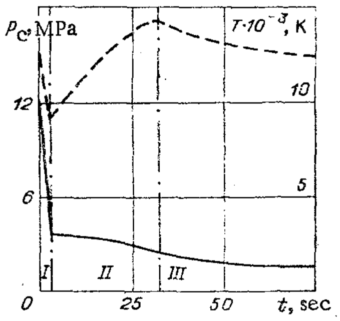

Fig. 2.

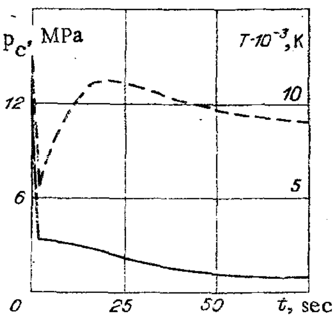

Fig. 3.

Infiltration in a Coarse Medium. In this case, the temperatures of the pieces of rock and the liquid do not equalize during the infiltration into the heated layer. Then the heat flux carried by the liquid is dependent on the size and shape of the pieces. We assume that the pieces are spherical, and then $E_t(t)$ can be determined by calculating the heat flux provided by the pieces to the liquid:

$$E_T(t) = -SN\int_0^t \lambda\nabla T|_{r=R_0}d\tau, \tag{22}$$

where $S = 4\pi R_0^2$ is the surface area of a piece; $N = V\Big/\frac{4}{3}\pi R_0^3$, number of pieces in the heated layer; V, volume of this layer; λ, thermal conductivity of the rock; and $\nabla T|_{r=R_0}$, temperature gradient at the boundary of a piece.

The quantity $\lambda\nabla T|_{r=R_0}$ is determined approximately by solving the one-dimensional problem for the cooling of a body whose surface is maintained at a constant temperature [6]:

$$-\lambda\nabla T|_{r=R_0} = (T_i - T^+)\frac{\lambda}{2\sqrt{\varkappa t}}, \tag{23}$$

where T_i is the initial temperature of a piece and T^+ is the temperature of the liquid outside the heated layer. We substitute (23) into (22) and use the fact that $T^+ \ll T_i$ to get

$$E_T(t) = (3E_1/R_0)\sqrt{\varkappa t}, \tag{24}$$

where E_1 is the energy localized in the heated layer. If $E_t(t)/E_0 \ll 1$, the value of $E_t(t)$ can be neglected in (6), (9), and (21), which is true for the times that satisfy

$$t \ll \frac{1}{9}\left(\frac{E_0}{E_1}\right)^2\frac{R_0^2}{\varkappa}.$$

We put $\varkappa = 1.4\cdot 10^{-3}$ cm²/sec, $R_0 \sim 5$ cm, $E_0/E_1 \sim 2$ to get $t \ll 10^4$ sec; we see that in this approximation (21) contains the similarity parameter $[k(p_0 - p_c)/(\mu^* R_c)]\sqrt{t/a}$, and equations (1)-(10) also satisfy this similarity law subject to (11).

Figure 3 shows results obtained with the same parameters. It was assumed that $R_0 = 5$ cm. Figure 3 shows that the time dependence of the temperature and pressure is qualitatively as for the finely divided medium. The pressure in the cavity is less than that in the first case, as would be expected.

Therefore, no matter what the grain size of the medium, the implosion results in a marked reduction in the temperature and a nonmonotone variation in the pressure. In the first stage, the reduction in cavity pressure is due to the temperature fall arising from the injection of the liquid, whose temperature is well below the temperature of the vapor of the refractory component. As L_1 (the latent heat of evaporation of the refractory component) is much more than L_2 (the latent heat of evaporation of water), the heat released by condensation of the refractory component is sufficient to evaporate and heat a mass of water large by comparison with the mass of the refractory component condensing: $m_{H_2O} \approx (L_1/L_2)m_{SiO_2}$, which results in an increase of the partial pressure of the water vapor in the cavity. After the condensation of most of the refractory component, further injection of the relatively cool liquid results in fresh falls in the temperature and pressure.

The nonmonotone pressure change is due mainly to the production of a considerable amount of heat on condensation of the refractory component. These calculations show that the heat transfer from the medium to the entering liquid (in the finely divided and coarsely divided media) does not qualitatively affect the changes in pressure and temperature. One can estimate the heat flow from the cavity by thermal conduction by using the solution to the one-dimensional problem for the cooling of a body whose surface is maintained at a constant temperature, namely the melting point of the refractory component. From (23) we have

$$Q(t) \approx T_i \frac{\lambda \sqrt{t}}{\sqrt{\varkappa}} 4\pi R_c^2. \tag{25}$$

The energy transported by the liquid from the heated layer into the cavity up to time t is

$$E_T(t) \approx T_i \frac{\lambda \sqrt{t}}{\sqrt{\varkappa}} 4\pi R_0^2 N. \tag{26}$$

From (25) and (26) we get the ratio of the fluxes as $Q(t)/E_t(t) \approx R_0 D/3R_c$, where R_c/D is the characteristic dimension of the heated region. As $D \approx 50$ and $R_c/D \gg R_0$ we have $Q(t)/E_t(t) \ll 1$, and Q(t) can be neglected.

These time curves for the pressure and temperature in the cavity apply up to the instant when the cavity collapses. In the case of a coarse medium, there is a similarity parameter $[k(p_0 - p_c)/\mu^* R_c]\sqrt{t/a}$ which enables one to derive the results for explosions of different energy outputs.

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